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Microscopic insight into nuclear structure properties of proton-rich barium isotopes

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Abstract Variation after projection (VAP) calculations in conjunction with Hartree Bogoliubov (HB) ansatz have been carried out for A=120–136 barium isotopes. In this framework, the yrast spectra with $I^{\pi}_{max} = 10^{+}$, B(E2) transition probabilities, quadrupole (β_2) and hexadecapole (β_4) deformation parameters for even- even barium isotopes have been obtained. The results of the calculation give an indication that it is important to include the hexadecapole-hexadecapole component of the two-body interaction for obtaining various nuclear structure quantities in these barium isotopes.

Keywords Nuclear Structure $^{120-136}$Ba, variation - after- projection calculations, B(E2) transition probabilities, quadrupole (β_2) and hexadecapole (β_4) deformation parameters

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1. Introduction

Over the years, there has been a continuous accumulation of extensive experimental information on proton-rich barium isotopes [1-8]. These proton-rich nuclei seem to develop a clear rotational structure in the light isotopes. An interesting feature in the observed yrast spectra in proton-rich barium isotopic mass chain in the systematic variation of E_2^+, E_4^+ and E_6^+ excitation energy states from ^{120}Ba to ^{136}Ba. It is observed that these states follow a systematic decreasing trend as we move away from ^{136}Ba towards ^{120}Ba. The isotopes $^{120-128}$Ba can be taken to be quasi-deformed nuclei having E_4^+/E_2^+ ratio larger than 2.7. In fact, this ratio for ^{120}Ba is 2.956 and for ^{128}Ba, is 2.686. Based on the systematics of low-lying states and the experimental data of B(E2) transition probabilities, the stable proton-rich barium isotopes range from the approximately spherical ^{138}Ba with a closed shell of N = 82 to ^{130}Ba which is close to the deformed $^{120-128}$Ba isotopes and these barium isotopes are thus said to be transitional.

The theoretical attempts [9-26] to describe the nuclear structure of barium isotopes include boson expansion theories [12, 13] and microscopic description of spherical nuclei, incorporating an harmonic effects [14, 15]. The description in terms of the shape parameters β and γ fall into two main groups - namely those that assume or predict these nuclei to be γ soft [16-19] and those that treat them as good rotors. These geometric theories generally favour prolate shapes for the barium isotopes. The interacting boson model [24,25] impressively reproduces the low-lying energy level systematics of only the stable even-mass barium isotopes. Recently, Petkov *et al* [26] have applied the general collective model to the barium isotopes with A = 124–132. A good description of the experimental level schemes is obtained. The description of the E2 transition strengths is also satisfactory but the deviations observed for the specific weak transitions indicate some crudeness in the model. Recently, Saha Sarkar and Sen [11] carried out Cranked Hartree-Bogoliubov (CHB) calculations in the Xe-Ba region. The properties of the ground state and first 2^+ state of a large number of even-even nuclei is obtained by them in this mass region.

From the overview of the theoretical work, it is evident that there is no single microscopic framework that has been applied uniformly to the entire set of barium isotopes with a single set of input parameters. It is with this motivation in mind that we plan to carry out a study of low-lying yrast spectra in the entire set of the proton-rich barium isotopic mass chain in a suitable microscopic framework.

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Thus in this paper, a microscopic study of the yrast states, B(E2) transition probabilities, quadrupole (β_2) and hexadecapole (β_4) deformation parameters, for even-even proton-rich barium isotopes has been carried out, by employing the variation after projection (VAP) [22] formalism in conjunction with the Hartree-Boboliubov (HB)[23] ansatz for the axially symmetric intrinsic wave-functions.

In the present variational calculations in proton-rich barium isotopes, we have employed the usual pairing plus quadrupole-quadrupole model of interaction (VQP) but we found that the level of agreement is poor with the experiments In order to improve the level of agreement, the higher order octupole-octupole and hexadecapole-hexadecapole terms (VQPOH) were also included in the two body interaction. At this point, we want to make it clear that addition of octupole-octupole interaction term to the VQP interaction does not make any perceivable change in the calculated values of the various observable quantities We have, therefore, skipped the presentation of these results. However, the inclusion of hexadecapole-hexadecapole interaction term to the VQP interaction shows a marked improvement in the various observable quantities.

2. Calculational details

2.1 The one and two-body parts of the Hamiltonian :

The spherical single-particle energies (S.P.E.'s) that we have employed are (in MeV) : $(2d_{5/2}) = 0.0$, $(3s_{1/2}) = 1.4$, $(2d_{3/2}) = 2.0$, $(1g_{7/2}) = 2.5$, $(1h_{11/2}) = 4.0$, $(2f_{7/2}) = 10.9$, $(1h_{9/2}) = 11.5$, $(1i_{13/2}) = 13.5$. The S.P.E.'s of $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, $1g_{7/2}$ and $1h_{11/2}$ are exactly the same as that employed by Vergados and Kuo [27] as well as Federman and Pittel [28]. The S.P.E.'s of $2f_{7/2}$, $1h_{9/2}$ and $1i_{13/2}$ orbits are taken from Nilsson diagrams, published in the book [29] with small variations so as to reproduce shell closures for N = 82 for ^{138}Ba.

The two-body effective interaction that has been employed is of pairing - plus-quadrupole-quadrupole ($q.q$) type [31]. The pairing part can be written as

$$V_p = -(G/4)\sum_{\alpha\beta} S_\alpha S_\beta a^\dagger_\alpha a^\dagger_{\bar\alpha} a_{\bar\beta} a_\beta , \qquad (1)$$

where α denotes the quantum numbers (nljm). The state $\bar\alpha$ is the same as α, but with the sign of 'm' reversed. Here, S_α is the phase factor $(-1)^{l-m}$. the q.q part of the interaction is given by

$$V_{qq} = -(\chi/2)\sum_{\alpha\beta\gamma\delta}\sum_{\mu}\langle\alpha| \qquad \beta|q^2_{-\mu}$$

$$\times(-1)^\mu a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma , \qquad (2)$$

where the operator q^2_μ is given by

$$q^2_\mu = (16\pi/5)^{1/2} r^2 Y^2_\mu(\theta,\phi) \qquad (3)$$

The strengths for the like particle neutron-neutron (n–n), proton-proton (p–p) and neutron-proton (n–p) components of the quadrupole-quadrupole ($q.q$) interaction were taken as

$$\chi_{nn}(=\chi_{pp}) = -0.0122 \text{ MeV } b^{-4} \text{ and}$$

$$\chi_{np} = -0.0230 \text{ MeV } b^{-4}$$

Here $b(=\sqrt{\hbar/m\omega})$ is the oscillator parameter. These values for the strengths of the $q.q.$ interactions compare favorably with the ones suggested by Arima [32] and these values are very near the ones employed in earlier study of deformation systematics in the $A \cong 100$ region [33-35]. The strength for the pairing interaction was fixed through the approximate relation $G = (18\text{-}21/A)$. In addition we have also carried out calculations by incorporating octupole-octupole and hexadecapole-hexadecapole interaction terms in the two-body interaction The relative magnitudes of the parameters of the octupole-octupole and hexadecapole-hexadecapole parts of the two body interaction were calculated from a relation suggested by Bohr and Mottelson [30]. The values of these parameters for octupole-octupole interaction work out to be

$$\chi_{pp3}(=\chi_{nn3}) = -0.00366 \text{ MeV } b^{-6}$$

$$\text{and } \chi_{pn3} = -0.00690 \text{ MeV } b^{-6}$$

whereas their values for hexadecapole-hexadecapole part of the two body interaction turn out to be

$$\chi_{pp3}4(=\chi_{nn4}) = -0.00033 \text{ MeV } b^{-8}$$

$$\text{and } \chi_{pn4} = -0.00066 \text{ MeV } b^{-8}$$

2.2 Projection of the states of good angular momentum from axially symmetric HB intrinsic states :

The procedure for obtaining the axially symmetric HB intrinsic states has been discussed in Ref. [36].

The axially symmetric HB states can be written as

$$|\Phi_0\rangle = \prod_{im}\left(u^m_i + v^m_i b^\dagger_{im}\, b^\dagger_{\bar{i}m}\right)|0\rangle , \qquad (4)$$

where the creation operators $b^\dagger_{im}$ can be expressed as

$$b^\dagger_{im} = \sum_j c^m_{ji} a^\dagger_{jm},\ b^\dagger_{\bar{i}m} = \sum_j (-1)^{l+j-m} c^m_{ji} a^\dagger_{j-m}. \qquad (5)$$

Here, the creation operators $a^\dagger_{jm}$ create a particle in the orbit $|nljm\rangle$ and c^m_{ji} are the expansion coefficients. The index j labels the single-particle state and the index i is employed to distinguish between the different deformed single-particle states with the same $\langle \hat{j}_z \rangle = m$.

The states with good angular momenta projected from the HB state $|\Phi_K\rangle$ can be written as

$$\psi^J_K >= P^J_{KK}|\Phi_K >= \left[(2J+1)/8\pi^2\right]\int D^J_{KK}(\Omega)\, R(\Omega)\,|\Phi_K > d\Omega\rangle, \quad (6)$$

where $R(\Omega)$ and $D^J_{KK}(\Omega)$ are the rotation operators of the rotation matrix, respectively.

The energy of the state with the angular momentum is given as

$$E_J = \left\langle \Phi_0 \middle| HP^J_{00} \middle| \Phi_0 \right\rangle / \left\langle \Phi_0 \middle| P^J_{00} \middle| \Phi_0 \right\rangle = \frac{\int_0^\pi h(\theta)\, d^J_{00}(\theta)\, d(\cos\theta)}{\int_0^\infty n(\theta)\, d^J_{00}(\theta)\, d(\cos\theta)}. \quad (7)$$

Here, the overlap integals $h(\theta)$ and $n(\theta)$ are given by

$$h(\theta) = n(\theta)\left[\sum_\alpha \varepsilon(\alpha)\rho_{\alpha\alpha} - (G/4)\sum_{\alpha\tau_3} S_\alpha \left[f(1+M)^{-1}\right]^{\tau_3}_{\alpha\bar{\alpha}}\right.$$

$$\left. \sum_{\beta\tau_3} S_\beta \left[(1+M)^{-1}F\right]^{\tau_3}_{\beta\beta} - (1/2)\sum_{\tau_3\tau'_3\mu}(-1)^\mu \chi_{\tau_3\tau'_3}\right.$$

$$\left. \sum_{\alpha\gamma} < \alpha \left| q^2_\mu \right| \gamma > \rho^{\tau_3}_{\alpha\gamma} \sum_{\beta\delta} < \beta \left| q^2_{-\mu} \right| \delta > \rho^{\tau_3}_{\delta\beta} \right. \quad (8)$$

where

$$n(\theta) = \left\{\det[1+M(\theta)]\right\}^{1/2}, \quad (9)$$

$$F_{\alpha\beta}(\theta) = \sum_{m_\alpha, m_\beta} d^J_{\alpha\, m_\alpha m_{\alpha'}}(\theta)\, d_{j_\beta m_\beta m_{\beta'}}(\theta)\, f_{j_\alpha m_{\alpha'}, j_\beta m_\beta}, \quad (10)$$

$$f_{\alpha\beta=\sum_\tau \nu_{\alpha\tau} j_{\alpha\tau}\, \nu_{\beta\tau} j_{\beta\tau}} \left(v^{m_\alpha}_i / u^{m_\alpha}_i\right)\delta_{m_\alpha, -m_\beta}, \quad (11)$$

$$M(\theta) = F(\theta) f^+, \quad (12)$$

$$\rho_{\alpha\beta}(\theta) = \left\{M(\theta)/[1+M(\theta)]\right\}_\alpha = \delta_{\alpha\beta} - \left\{[1+M(\theta)]^{-1}\right\}_{\alpha\beta}. \quad (13)$$

The yrast energies are calculated as follows. Using the results of the HB calculations summarized in terms of the amplitudes $\left(u^m_i, v^m_i\right)$ and the expansion coefficients c^m_μ, we first set up the (50 × 50) f-matrix in the present configuration space. Then F, M, and $(1+M)^{-1}$ are computed for 20 Gaussion quadrature points in the range $(0, \pi/2)$. Finally, the projected energies are calculated employing eqs. (7) – (13).

2.3 The variation-after-angular momentum projection (VAP) method :

The Variation After Projection (VAP) calculations has been varried out as follows. We first generated the self-consistent HB solutions $\Phi(\beta)$ by carrying out the HB calculations with the Hamiltonian $(H - \beta Q^2_0)$, where β is a parameter. The selection of the optimum intrinsic states $\Phi_{opt}(\beta_J)$ is then made by finding out the minimum of the projected energy.

$$E_J(\beta) = \left\langle \Phi_0(\beta) \middle| HP^J_{00} \middle| < \Phi_0(\beta) > / < \Phi_0(\beta) > \middle| P^J_{00} \middle| \Phi_0(\beta) \right\rangle \quad (14)$$

as a function of β. In other words, the optimum intrinsic state for each yrast J satisfies the conditions

$$\partial / \partial\beta \left[\left\langle \Phi_0(\beta) \middle| HP^J_{00} \middle| \Phi_0(\beta) \right\rangle / \left\langle \Phi_0(\beta) \middle| P^J_{00} \middle| \Phi_0(\beta) \right\rangle\right]_{\beta=\beta J} = 0 \quad (15)$$

3. Deformation trends in the barium region

In Table 1, the experimental values of excitation energy of E^+_2 state (ΔE), intrinsic quadrupole moments of the HB states and quadrupole deformation parameters (β_2) obtained with the interaction VQP in $^{120\text{-}136}$Ba istopes are presented. It is well known that a nucleus having smaller energy gap ΔE should have a larger quadrupole moment for the $2_1{}^+$ state. Since $Q_2{}^+$ of the nucleus is directly related to its intrinsic quadrupole moment, one should, therefore, expect that a smaller energy gap ΔE should manifest itself in terms of a larger quadrupole moment and vice-versa. In other words, the observed systematics of $E_2{}^+$ with mass number A should produce a corresponding inverse systematics of intrinsic quadrupole moment of barium nuclei with A. Based on the above logic, the calculated values of intrinsic quadrupole moments should exhibit a decreasing trend as one moves from ^{120}Ba to ^{136}Ba. This type of trend is certainly should be the calculated values of $<Q^2_0>_{HB}$ obtained with the interaction VQP. The changes in $<Q^2_0>_{HB}$ values with VQP is 1.90 units as we move from ^{120}Ba to ^{126}Ba whereas the $E_2{}^+$ energy value is seen to rise from 0.18 MeV to 0.25 MeV. Same type of trend is shown by the calculated values of $<Q^2_0>_{HB}$ with VQPOH interaction with a difference that it shows a slightly larger change as one moves from one isotope to another as compared to the $<Q^2_0>_{HB}$ values obtained with VQP. The changes in $<Q^2_0>_{HB}$ values with VQPOH are 3.44 units as we move from ^{120}Ba to ^{126}Ba. Since the experimental $E_2{}^+$ values vary from isotope to isotope, the results of $<Q^2_0>_{HB}$ obtained with VQPOH can be

Table 1. The experimental values of excitation energy of the E_2^+ state in the MeV (ΔE), intrinsic quadrupole moments and quadrupole deformation parameters (β_2) of the HB states in $^{120-136}$Ba isotopes with the VQP and VQPOH interaction. Here $<Q_0^2>_\pi$ gives the contribution of the protons to the total intrinsic quadrupole moments. The quadrupole moments have been computed in units of b^2, where $b=\sqrt{\hbar/m\omega}$ is the oscillator parameter

Barium nuclei (A)	E_2^+ (Exp.)	Interaction: VQP $<Q_0^2>_{HB}$	VQP $<Q_0^2>_\pi$	VQP β_2	VQPOH $<Q_0^2>_{HB}$	VQPOH $<Q_0^2>_\pi$	VQPOH β_2	β_2 (Exp.)
120	0.18	93.56	35.20	0.297	93.70	35.58	0.298	–
122	0.19	93.46	35.10	0.291	93.41	34.50	0.290	–
124	0.22	92.75	35.08	0.281	91.55	34.66	0.280	0.295(6)**
126	0.25	91.66	34.99	0.272	90.26	33.06	0.271	0.285(16)*
128	0.28	86.54	33.98	0.259	80.30	28.77	0.237	0.238(10)*
130	0.35	74.85	30.03	0.221	73.87	28.77	0.218	0.230(12)*
132	0.46	65.08	28.66	0.193	64.34	28.61	0.191	0.186(6)*
134	0.60	57.64	28.42	0.173	58.47	28.25	0.174	0.163(19)*
136	0.81	50.59	28.02	0.153	50.37	27.82	0.15	0.124(8)*

*Data taken from Ref. [5]

**Data taken from Ref. [42]

thought to be in better agreement than the corresponding results obtained with VQP. In the same table, the calculated values for deformation parameter (β_2) have also been presented. The deformation parameter β_2 is related to $B(E2)\uparrow$ by the formula suggested by Raman *et al* [37].

$$\beta_2 = (4\pi/3ZR_0^2)\left[B(E2)\uparrow/e^2\right]^{1/2}, \qquad (16)$$

where R_0 is usually taken to be $1.2A^{1/3}$ fm and $B(E2)\uparrow$ is in units of e^2b^2.

From the systematics of the calculated β_2 values, it is noted that β_2 values obtained with VQP and VQPOH interactions are in satisfactory agreement with the observed values.

We next focus our attention on the factors that are responsible for making these barium isotopes to exhibit such features. It is important at this stage to point out and highlight an important factor responsible for bringing in collectivity in nuclei. Whenever the occupation probability of a low k-component of a high j- orbital decreases (increases), it produces large decrease (increase) in intrinsic quadrupole moments. Against this back-ground, a comparison of Tables 2 and 4 shows that the decrease in $(h_{11/2})_\pi$ occupation obtained with VQP is small compared to the decrease shown by $(h_{11/2})_\pi$ values

Table 2. The sub-shell occupation numbers (protons) in the nuclei $^{120-136}$Ba with VQP interaction

Barium nuclei (A)	Sub-shell occupation number: $3s_{1/2}$	$2d_{3/2}$	$2d_{5/2}$	$2f_{7/2}$	$1g_{7/2}$	$1h_{9/2}$	$1h_{11/2}$	$1i_{13/2}$
120	0.61	0.99	1.30	0.50	1.11	0.00	1.47	0.00
122	0.61	0.98	1.30	0.52	1.10	0.01	1.46	0.00
124	0.61	0.98	1.31	0.51	1.10	0.00	1.46	0.00
126	0.61	0.99	1.32	0.50	1.11	0.00	1.45	0.00
128	0.60	1.05	1.48	0.41	1.20	0.00	1.25	0.00
130	0.59	1.24	2.13	0.10	1.50	0.00	0.40	0.00
132	0.57	1.35	2.49	0.00	1.57	0.00	0.00	0.00
134	0.56	1.34	2.61	0.00	1.47	0.00	0.00	0.00
136	0.54	1.33	2.77	0.00	1.34	0.00	0.00	0.00

Table 3. The sub-sheel occupation numbers (neutrons) in the nuclei $^{120-136}$Ba with VQP interaction

Barium nuclei (A)	Sub-shell occupation number: $3s_{1/2}$	$2d_{3/2}$	$2d_{5/2}$	$2f_{7/2}$	$1g_{7/2}$	$1h_{9/2}$	$1h_{11/2}$	$1i_{13/2}$
120	0.60	1.38	3.25	1.05	2.75	0.06	4.88	0.00
122	0.87	1.81	3.65	1.08	3.55	0.06	4.85	0.11
124	0.91	1.88	4.17	1.07	5.01	0.06	4.85	0.00
126	0.92	1.88	4.20	1.04	4.99	0.06	4.89	0.00
128	0.93	1.88	5.55	1.09	5.49	0.11	6.79	0.12
130	1.74	2.34	5.81	0.94	5.72	0.12	7.14	0.15
132	1.99	3.36	5.94	0.80	6.22	0.08	7.46	0.11
134	1.99	3.82	5.98	0.72	7.07	0.08	8.33	0.00
136	1.99	4.00	6.00	0.63	8.00	0.06	9.30	0.00

obtained with VQPOH, for the set of $^{120\text{-}126}$Ba isotopes. In the first case the decrease is 0.02 whereas in the second case the decrease is 0.82. The decrease in $(h_{11/2})_\pi$ occupation number should be associated with a decrease in the intrinsic quadrupole moments $< Q_0^2 >_\pi$ which will in turn manifest itself in terms of the increase in the observed E_2^+ energy. The type of increase in energy in $^{120\text{-}126}$Ba is seen to be commensurate with the type of decrease in $(h_{11/2})_\pi$ occupation numbers presented in Table 4, as here a depletion in the down sloping $k = 1/2$ component of $(h_{11/2})_\pi$ orbit is taking place. After ^{126}Ba the occupation is decreasing gradually to zero as seen from Table 2, whereas this quantity goes to zero faster as is exhibited by the results presented in Table 4. A comparison of results on neutron

Table 4. The sub shell occupation numbers (protons) in the nuclei $^{120\text{-}136}$Ba with VQPOH interaction

Barium nuclei (A)	Sub-shell occupation number							
	$3s_{1/2}$	$2d_{3/2}$	$2d_{5/2}$	$2f_{7/2}$	$1g_{7/2}$	$1h_{9/2}$	$1h_{11/2}$	$1i_{13/2}$
120	0 46	0 77	1 26	0 67	0 99	0 02	1 79	0 00
122	0 48	0 81	1 31	0.64	1 05	0 01	1 66	0 00
124	0 52	0 87	1 39	0 54	1 12	0 00	1 51	0 00
126	0 63	1 14	1 62	0 35	1 27	0 00	0 97	0 00
128	0 66	1 51	2 31	0 00	1 51	0 00	0 00	0 00
130	0 61	1 31	2 11	0 10	1.85	0 00	0 00	0 00
132	0 59	1 39	2 50	0 00	1.50	0 00	0 00	0 00
134	0 61	1 46	2 60	0 00	1 31	0 00	0 00	0 00
136	0 58	1 43	2 78	0.00	1 18	0 00	0 00	0 00

occupation numbers shows that $(h_{11/2})_\nu$ occupation becomes more than half for $^{126\text{-}136}$Ba. Both these effects are responsible for large decrease in quadrupole moments. Further, for the set of $^{120\text{-}136}$Ba isotopes, the results presented in Tables 3 and 5 indicate that the sub-sells $(d_{5/2})_\nu$, $(g_{7/2})_\nu$ and $(h_{11/2})_\nu$ are becoming more than half-filled and heading towards shell closures with increasing neutron number. The effect of all this is seen in terms of decreasing quadrupole moments with large changes as one moves from ^{128}Ba to ^{136}Ba. From the results of occupation probabilities presented in Tables 2, 3, 4 and 5, the overall picture that emerges is that the results obtained with VQPOH interaction on occupation probabilities presented in Tables 4 and 5 are found to be in better harmony with the observed E_2^+ systematics. Further, one can conclude that the observed systematics in $^{120\text{-}136}$Ba isotopes are intimately dependent on the manner in which $(h_{11/2})_\pi$ and $(h_{11/2})_\nu$ occupation probabilities change. For $^{120\text{-}126}$Ba, $(h_{11/2})_\pi$ occupation probabilities decrease and for $^{128\text{-}136}$Ba the $(h_{11/2})_\nu$ become increasingly more than half-filled

Table 5. The sub-shell occupation numbers (neutrons) in the nuclei $^{120\text{-}136}$Ba with VQPOH interaction

Barium nuclei (A)	Sub-shell occupation number							
	$3s_{1/2}$	$2d_{3/2}$	$2d_{5/2}$	$2f_{7/2}$	$1g_{7/2}$	$1h_{9/2}$	$1h_{11/2}$	
	0 44	1 06	3 21	1.01	2.95	0 14	4 84	0 31
122	0 90	1 53	3.54	1 01	3 60	0.13	4 84	0 40
124	0 90	1 63	4.59	0.99	4 58	0 11	4.88	0.28
126	0 90	1 86	4.18	1.17	5.03	0 12	6 69	0.00
128	0 97	1 90	5.55	0.96	5.43	0 07	6 89	0 12
130	1 68	2.51	5.82	1.01	5 74	0.11	6 90	0 19
132	1 98	3.39	5 95	0.79	6 31	0.07	7.56	0.00
134	1 99	3.65	5 97	0.74	6.78	0 07	8.69	0 08
136	1 99	3 99	5.99	0.63	7.99	0.05	9.30	0.00

4. Transition probabilities

The reliability and goodness of the HB wave-function is examined by calculating the $B(E2; 0_1^+ \rightarrow 2_1^+)$ values. Sometimes back Bhatt *et al* [38] have developed formula for the calculation of $B(E2, 0_1^+ \rightarrow 2_1^+)$ transition probabilities from the values of intrinsic quadrupole moments of protons and neutrons, in the mass region of rare earth and actinide nuclei. It has been justified by them that the $B(E2; 0_1^+ \rightarrow 2_1^+)$ in units of e^2b^2 are given by

$$B(E2, 0_1^+ \rightarrow 2_1^+) = (1.02 \times 10^{-5}) A^{2/3} c^2_{\text{model}} \left[e_\pi \left\langle Q_0^2 \right\rangle_\pi + e_\nu \left\langle Q_0^2 \right\rangle_\nu \right]^2, \quad (17)$$

where $< Q_0^2 >_\pi$ ($< Q_0^2 >_\nu$) are the intrinsic quadrupole moments of valence protons (neutrons) and e_π and e_ν are the effective charges of the protons and neutrons respectively.

Effective charges of the protons and neutrons are given by

$$e_\pi = (1 + Z/A)e \quad \text{and} \quad e_\nu = \varepsilon(Z/A)e .$$

They have recommended the use of $\varepsilon = 2.1$ and $c_{\text{model}} = (0.8 \pm 0.07)$ in the rare earth and actinide regions. We have used this formula for the calculation of the B(E2) values for the mass chain of $^{120\text{-}136}$Ba isotopes.

In Table 6, we present a comparison of the calculated B(E2) values obtained with VQP and VQPOH interaction with the experimental values for the $0_1^+ \rightarrow 2_1^+$ transitions in case of $^{120\text{-}136}$Ba. It is satisfactory to note that the calculated B(E2) values are in satisfactory agreement with the experimental values for the $0_1^+ \rightarrow 2_1^+$ transition in case of $^{120\text{-}13}$Ba. Since the calculations for the B(E2) values depend on the intrinsic quadrupole moments so B(E2) values should follow the same trend as that followed by the intrinsic quadrupole moments. This feature of the barium isotopes has been reproduced by the present calculations. Based on the calculated results of B(E2)

Table 6. Comparison of the calcualted and experimental $B(E2;0_1^+ \quad 2_1^+)$ values in $^{120-136}Ba$ isotopes The B(E2) values are in units of e^2b^2

Barium nuclei (A)	$B(E2;0_1^+ \rightarrow 2_1^+)$		
	Interaction		
	VQP	VQPOH	(Exp)
120	1 99	1 99	-
122	1 96	1 95	-
124	1.91	1 87	-
126	1 79	1 77	1 91(21)*
128	1 65	1 38	1 36(11)*
130	1 23	1 19	1 29(14)*
132	0 96	0 94	0 86(6)*
134	0 78	0 79	0 68(16)*
136	0 63	0 62	0.53(16)**

* Data taken from Ref [5]
** Data taken from Ref [43]

values, it seems as if the wave-functions obtained with VQP and VQPOH are equally reliable for the $^{120-136}Ba$.

5. Yrast spectra

A projection calculation for the energy spectra of $^{120-136}Ba$ was carried out by employing the VQP and VQPOH models of two body interaction. In Figures 1, 2 and 3, the yrast spectra for $^{120-136}Ba$ isotopes is displayed. In Figures 1, 2, and 3, we have compared the experimental values of yrast states with the theoretical values (Th.1 and Th.2). The spectra corresponding to Th. 1 has been obtained by using VQP two body interaction where as the spectra corresponding to Th. 2 has been obtained by including a higher order octupole-octupole and hexadecapole-hexadecapole interaction energy terms in the two body residual interaction (VQPOH). It turns out from our calculations that calculated spectra corresponding to Th 2 reproduces the observed yrast spectra for $^{120-136}Ba$ reasonably well as compared to the yrast spectra corresponding to Th 1

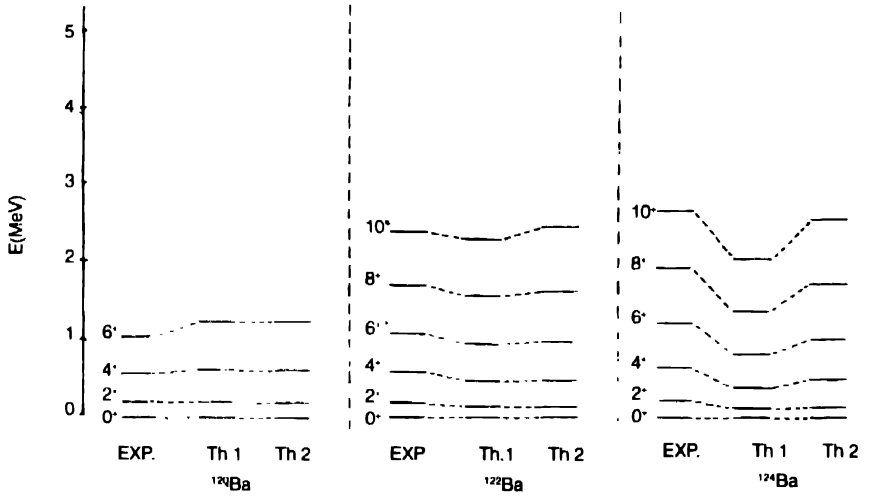

Figure 1. Experimental and theoretical low-laying yrast spectra for $^{120-124}Ba$ nuclei (Data taken from Ref [6])

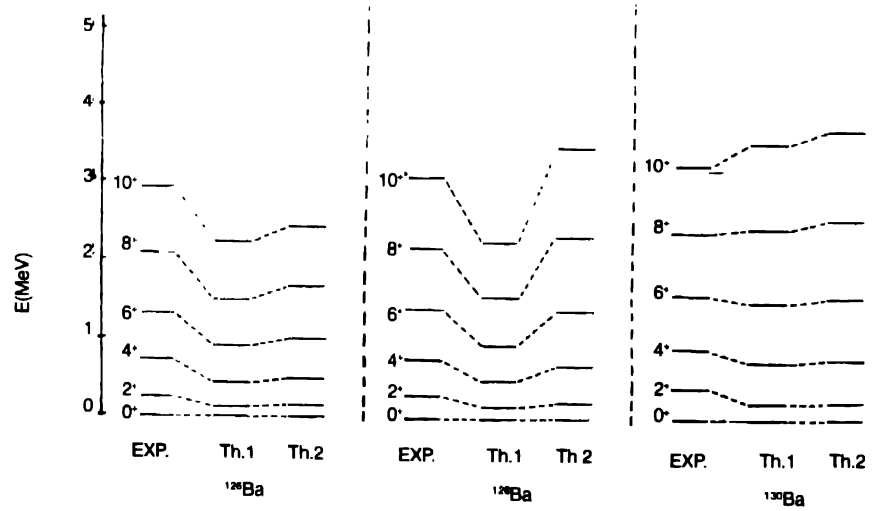

Figure 2. Experimental and theoretical low lying yrast spectra for $^{126-130}Ba$ nuclei. (Data taken from Ref. [6])

This level of agreement can be considered to be satisfactory because of a number of considerations. First, the calculation of yrast spectra is a complex many body calculation involving a calculated spectra with the experiments. We have checked that there is very little difference occuring in the yrast spectra when octupole-octupole interaction term is added to VQP interaction.

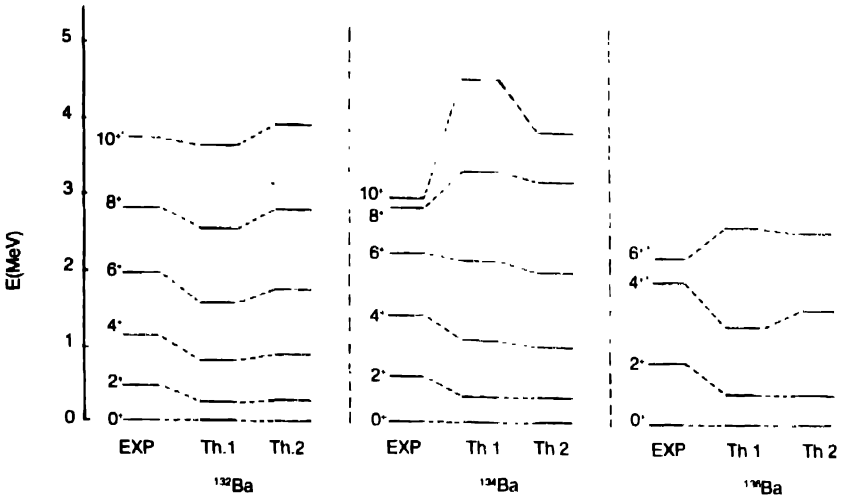

Figure 3. Experimental and theoretical low lying yrast spectra for ^{132,134,136}Ba nuclei (Data taken from Ref [6]).

minimum of 20 valence particles for ^{120}Ba and a maximum of 36 valence particles for ^{136}Ba. Another noticeable fact is that the calculations are carried out for the entire set of the $^{120\text{-}136}$Ba isotopes, with a single set of input parameters. Also, no parameter has been used to simulate the contribution of the tin core towards the moment of inertia. It was pointed out by Faessler [39] as well as by Praharaj [40] that doing so, will provide further quantitative improvements in the calculated yrast spectra. Thus, from the figures, it is clear that the inclusion of the hexadecapole-hexadecapole interaction improves the agreement of the It was observed by us that inclusion of only octupole-octupole component of two – body residual interaction did not make any noticeable difference to the calculated nuclear structure quantities.

6. Hexadecapole deformations (β_4)

In order to check the reliability of the hexadecapole strength parameters and the corresponding HB wave-function, the values of $<Q^4_0>$ and β_4 have been calculated using VQP and VQPOH interactions.

Table 7. Hexadecapole moments and hexadecapole deformation parameters (β_4) of the HB states in $^{120\text{-}136}$Ba isotopes with the VQP and VQPOH interaction. Here $<Q^4_0>_\pi$ ($<Q^4_0>_\nu$) give the contribution of the protons (neutrons) to the total intrinsic hexadecapole moments. The hexadecapole moments have been computed in units of b^4, where $b = \sqrt{\hbar/m\omega}$ is the oscillator parameter.

Barium nuclei	VQP				VQPOH				β_4*
(A)	$<Q^4_0>_{HB}$	$<Q^4_0>_\pi$	$<Q^4_0>_\nu$	β_4	$<Q^4_0>_{HB}$	$<Q^4_0>_\pi$	$<Q^4_0>_\nu$	β_4	
120	103 612	77.363	26.249	0 023	207 614	114 522	93 095	0.046	0.039
122	76 821	76.240	0 580	0.017	188 002	110 176	77 825	0 041	0 020
124	42.861	71.791	-28 930	0 009	-4 353	57 212	-61 565	-0 0009	-0 001
126	5 786	78 198	-72 411	0 001	-10 032	50 092	-60 062	-0 022	-0 016
128	-8.259	51.532	-59.792	-0 001	-80 664	-1 372	-79 292	-0 018	-0 014
130	-9.061	33 45	-42.51	-0.0019	-8.880	25 963	-34.847	-0 001	-0 008
132	-10.359	14 825	-25.184	-0 0021	-16 808	9 741	-26 550	-0 003	-0 012
134	-10.64	14.118	-24.765	-0.0022	-41 569	2.538	-44 107	-0.009	-
136	-15.234	13.371	-28.606	-0.0032	-32.675	3.190	-35.865	- 0 007	-

*Data taken from Ref. [41].

The results are presented in Table 7. It is observed that β_4 values obtained with VQPOH interaction are larger in magnitude than the β_4 values obtained with VQP interaction. Besides, the β_4 values with VQPOH interaction are seen to change their sign from positive to negative at ^{124}Ba whereas those with VQP show this change at ^{128}Ba. In the same table we have given the calculated values of β_4 obtained by Granderath *et al* [41] for $^{120\text{-}132}$Ba. It is noteworthy that the values of β_4 calculated with VQPOH interaction are in satisfactory agreement with the β_4 values calculated by Granderath. Further, the change in sign in the β_4 values is also reproduced at the same barium isotope. From this, we conclude that HB wave-function obtained with VQPOH interaction represents the states of $^{120\text{-}136}$Ba isotopes more accurately than that obtained with VQP interaction. In addition, it points out that it is important to include the hexadecapole interaction term in the two-body residual interaction for nuclear structure calculations of $^{120\text{-}136}$Ba isotopes.

7. Conclusions

From the results of our calculations, the following conclusions can be drawn.

(1) The VAP calculations performed with VQPOH interaction reproduces correctly the observed deformation systematics in $^{120\text{-}136}$Ba isotopes. The deformation systematics is seen to depend sensitively on the $(h_{11/2})_\pi$ occupation in case of $^{120\text{-}126}$Ba and $(h_{11/2})_\nu$ occupation in $^{128\text{-}136}$Ba.

(2) The yrast spectra obtained with inclusion of hexadecapole interaction shows satisfactory agreement with observed spectra as compared to the spectra obtained with VQP model of interaction.

(3) The values of hexadecapole interaction parameters employed by us are the appropriate one's in this mass region as with them, the HB wave-function yields values of B(E2), β_2 and β_4 which are in satisfactory agreement with experiments.

(4) The quadrupole-quadrupole plus pairing model of two-body interaction gives a better overal understanding of the properties of $^{120\text{-}136}$Ba isotopes.

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